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Covariant Quantization of The Super-D-string

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Abstract

We present the covariant BRST quantization of the super-D-string. The non-vanishing supersymmetric U(1) field strength \mathcal{F} is essential for the covariant quantization of the super-D-string as well as for its static picture. A SO(2) parameter parametrizes a family of local supersymmetric (kappa symmetric) systems including the super-D-string with $\mathcal{F} \neq 0$ and the Green-Schwarz superstring with $\mathcal{F} = 0$. We suggest that E^1 (canonical conjugate of U(1) gauge field) plays a role of the order parameter in the Green-Schwarz formalism: the super-D-string exists for $E^1 \neq 0$ while the fundamental Green-Schwarz superstring exists only for $E^1 = 0$.

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1 Introduction

D-branes play important roles to study non-perturbative aspects of superstring theories [1]. Especially the $SL(2, \mathbb{Z})$ duality found in the low energy effective theory of the type IIB superstring theory [2] relates the fundamental superstring and a series of soliton solutions as the $SL(2, \mathbb{Z})$ multiplet. The fact that super-D-brane states belong to the same multiplet as those of type II fundamental superstring tells they share the same symmetry structures. Intensive studies of dualities are establishing physical pictures of D-branes [1, 2, 3, 4]. The D-string (D1 brane) is required as the S dual object of the type IIB superstring and has properties: (1) it has the R-R charge, (2) it is the BPS saturated state, (3) its tension is scaled as a representation of $SL(2, \mathbb{Z})$, (4) it is consistent with the static gauge. The action for super-D-branes was written down in the manifest symmetric form under global space-time supersymmetry and τ, σ reparametrization and the local supersymmetry (kappa symmetry) [5, 6, 7, 8]. Since the guiding principle of construction of actions is the local symmetries, the super-D-string and the Green-Schwarz superstring have similar symmetry structures though their physical pictures are quite different. In this paper, we start with the super-D-brane action [5, 6, 7, 8] and examine how above physical pictures of D-branes are arisen.

There is also an interesting issue of covariant quantization of super-D-branes as space-time supersymmetric objects. Despite of simple description of the Green-Schwarz (GS) superstring [9] the covariant quantization without infinite number of fields has been a long standing problem. The origin of the difficulty of the covariant quantization of GS superstring is existence of massless point-like ground states. It is suggested that such massless ground states do not appear once we accept the super-D-brane picture [10]. This massiveness condition is satisfied if the static gauge were taken [6]. The real question is whether the static gauge can be chosen and the massiveness condition is satisfied for the D-brane states especially for the ground states. To answer it we rather begin by the action and clarify the condition for the static and massive picture of super-D-branes.

The physical picture of D-branes is the one of “fat strings” described by the partial spontaneous breaking of the global supersymmetry algebra [11]. A lying string breaks some of translational symmetries and supersymmetries, and it gives rise to goldstone coordinates and goldstinos respectively. In order to make Lorentz covariant action, extra coordinates associated with unbroken symmetry generators are introduced in the gauge invariant way, so that extra coordinates are set to be redundant. For the super-D-string system, covariance of the gauge, $\theta_1 = 0$, reflects covariance of the Nambu-Goldstone fermion, θ_2 . Due to the Wess-Zumino action [5, 6, 7, 8], the global supersymmetry algebra allows the central extension [1] which leads to the BPS condition representing the spontaneous symmetry breaking. Since the global supersymmetry algebra is similar to the local supersymmetry algebra, the central extension enables us to separate fermionic constraints in covariant and irreducible way. In this paper we confirm this beautiful consistency in the canonical language. We also reexamine the covariant gauge from the viewpoint of the equation of motion and the boundary condition.

The plan of this paper is the following: In section 2, we perform canonical analysis and

the BRST quantization of the super-D-particle. Although it has been examined in detail in [10] we review it for the later application. Especially the structure of the irreducible covariant separation of the first class and the second class constraints is clarified. The BPS bound is derived from the N=2 global supersymmetry algebra. One half of supersymmetry is realized as usual by using the Nambu-Goldstone fermion, and another half is realized rather trivially. This is explained in terms of the Dirac starred variables. In section 3, the super-D-string action is analyzed in the canonical formalism. The singularity which would prevent the covariant quantization is carefully examined. In section 4, the BRST charge of the super-D-string is constructed and the covariant gauge fixed actions are obtained. We clarified how the super-D-string overcomes difficulties of the covariant quantization of the superstring, such as how the ghost for ghost does not appear. The global supersymmetry algebra is calculated and the BPS property is explained. In section 5, we present a family of local supersymmetric actions parametrized by a SO(2) parameter and the relation between the Green-Schwarz superstring and the super-D-string is clarified. It is also noted that the general background makes the SUSY central charges SL(2,R) covariant representation.

2 Covariant quantization of the super-D-particle

The action of the super-D-particle is given by [6]

$$\begin{aligned} S &= -T \int d\tau \sqrt{-\det(G + \mathcal{F})} + T \int \Omega_{(1)} \\ &= \int d\tau T [-\sqrt{-\Pi_0^2} + \bar{\theta} \Gamma_{11} \dot{\theta}] \end{aligned} \quad (2.1)$$

where $\Pi_0^m = \dot{X}^m - \bar{\theta} \Gamma^m \dot{\theta}$ is the super symmetric velocity. The Majorana-Weyl spinors $\theta_A (A = 1, 2)$ have opposite chiralities; $\Gamma_{11} \theta_A = (-1)^{A-1} \theta_A$. Although the system is examined in some detail by Kallosh [10] we give the canonical analysis here with some remarks.

In the canonical formalism it follows the primary constraints

$$h \equiv \frac{1}{2T} (p^2 + T^2) = 0 \quad (2.2)$$

$$f \equiv \zeta + \bar{\theta} (\not{p} - T \Gamma_{11}) = 0 \quad (2.3)$$

where ζ_A , with $\zeta_A \Gamma_{11} = (-1)^{A-1} \zeta_A$, are canonical momenta conjugate to θ_A . The first one is the mass shell condition that the D-particle has its mass T . The second is fermionic constraints satisfying following algebra

$$\{f_{A,\alpha}, f_{B,\beta}\} = 2(C\Sigma)_{A,\alpha B,\beta} \quad (2.4)$$

with

$$\Sigma_{AB} = (\not{p} - T \Gamma_{11}) = \begin{pmatrix} -T & \not{p} \\ \not{p} & T \end{pmatrix} \quad (2.5)$$

Here the chiral representation is chosen such that Γ_{11} is diagonal and the charge conjugation matrix, C , and Γ_μ are off-diagonal. As in the case of massless superparticle, it is nilpotent on the constraint (2.2) as a matrix

$$\sum_{B=1,2} \Sigma_{AB} \Sigma_{BC} \approx 0 \quad . \quad (2.6)$$

However each block of the matrix Σ is not nilpotent so it has non-zero determinant in contrast with the superparticle case,

$$\det (\Sigma_{AB}) \neq 0 \quad . \quad (2.7)$$

Irreducible and covariant separation of fermionic constraints is possible by using (2.6) and (2.7). One half of eigenvalues is zero while another half is non-zero. In the rest frame $\Sigma \rightarrow T\Gamma_0(1 + \Gamma_0\Gamma_{11})$ where $(1 + \Gamma_0\Gamma_{11})/2$ is a projection operator. Hence the rank of Σ is one half of its maximum, $32/2$. Therefore one half of the fermionic constraints, $f_{A,\alpha}$, is the first class and another half is the second class. The fermionic first class constraints are

$$\tilde{f}_2 \equiv f\Sigma \frac{1 - \Gamma_{11}}{2} = f(-T\Gamma_{11})(1 - \gamma^{(0)}) \frac{1 - \Gamma_{11}}{2} = \zeta_2 \not{p} + T\zeta_2 + \bar{\theta}_1(p^2 + T^2) = 0 \quad , \quad (2.8)$$

where the projection factor $(1 - \gamma^{(0)})$ is the canonical version of the projection operator of Schwarz [6]. The second class constraints are

$$f_1 \equiv f \frac{1 + \Gamma_{11}}{2} = \zeta_1 + \bar{\theta}_1 \not{p} - T\bar{\theta}_2 = 0 \quad . \quad (2.9)$$

The algebra of first class constraints and second class constraints is calculated as;

$$\{\tilde{f}_2, \tilde{f}_2\} = -4TC\not{p}h \quad , \quad \{f_1, f_1\} = 2C\not{p} \quad , \quad \{\tilde{f}_2, f_1\} = -2TCh \quad . \quad (2.10)$$

The Dirac bracket for the second class constraints, $f_1 = 0$, is defined by

$$\{A, B\}_D = \{A, B\} - \{A, f_1\} \frac{\not{p}C^{-1}}{2p^2} \{f_1, B\} \quad . \quad (2.11)$$

The covariant separation of fermionic constraints into (2.8) and (2.9) enables us to construct the covariant path integral. The second class constraints are built in the path integral measure [13]:

$$Z = \int d\mu_{\text{can}} \delta(f_1) (\det\{f_1, f_1\})^{-1} \exp i \left[\int \left(p\dot{X} + \zeta_A \dot{\theta}_A + b\dot{c} + \beta_2 \dot{\gamma}_2 - H_{gf+gh} \right) \right] \\ d\mu_{\text{can}} = \mathcal{D}[\zeta_A, \theta_A, p, X; c, b, \gamma_2, \beta_2] \quad . \quad (2.12)$$

Where the gauge fixed and ghost part of Hamiltonian is

$$H_{gf+gh} = \{\Psi, Q_B\}_D \quad (2.13)$$

with the gauge fixing function, Ψ , and the BRST charge, Q_B .

The BRST charge is obtained as

$$Q_{B,\min} = hc + \tilde{f}_2 \gamma_2 + 2bT \tilde{\gamma}_2 \not{D} \gamma_2 \quad (2.14)$$

with nilpotent property; $\{Q_{B,\min}, Q_{B,\min}\}_D = 0$. Since we have irreducible constraints infinite number of ghosts are not required unlike the superparticle case. In order to perform gauge fixing, a bosonic auxiliary field and a fermionic auxiliary canonical pair ψ and π_ψ are introduced. The total BRST charge is given by

$$Q_B = Q_{B,\min} + \pi_g \tilde{c} + \pi_\psi \tilde{\gamma}_2 \quad (2.15)$$

with nilpotent property; $\{Q_B, Q_B\}_D = 0$.

Corresponding to the covariant gauge, $\theta_2 = 0$, the gauge fixing function is taken as,

$$\Psi = bg - \beta_2 \psi_2 - \tilde{\beta}_2 \theta_2 \quad (2.16)$$

Here the reparametrization gauge is kept unfixed in order to focus on fermionic gauge fixing procedure. Auxiliary field χ , η_1 and ρ_1 are introduced to exponentiate the contribution of second class constraints in the measure. Then the path integral becomes

$$Z = \int d\mu_{\text{can}} \mathcal{D}[\chi_1, \eta_1, \rho_1; g, \psi, \pi_\psi, \tilde{\gamma}_2, \tilde{\beta}_2] \exp i \int \mathcal{L} \quad (2.17)$$

$$\begin{aligned} \mathcal{L} = & f_1 \chi_1 + \bar{\eta}_1 \frac{\not{D}}{2T^2} \rho_1 + p(\dot{X} - \bar{\theta}_1 \Gamma \dot{\theta}_1) - gh + \tilde{f}_2 \psi_2 + \pi_\psi \theta_2 \\ & + b\dot{c} + b \frac{4}{T} \tilde{\gamma}_2 \not{D} \psi + \beta_2 \dot{\gamma}_2 + \beta_2 \tilde{\gamma}_2 + \tilde{\beta}_2 \gamma_2 \quad . \end{aligned} \quad (2.18)$$

Most variables can be integrated out and it ends up with just overall constant. There remain integrations over physical variables, X and θ_1 , and reparametrization ghosts, b and c :

$$Z = \int \mathcal{D}[X, p, \theta_1, g, b, c] \exp \left[i \int \left(p(\dot{X} - \bar{\theta}_1 \Gamma \dot{\theta}_1) - gh + b\dot{c} \right) \right] \quad (2.19)$$

$$= \int \mathcal{D}[X, \theta_1, g, b, c] \exp \left[i \int \left(-\frac{T}{2g} (\dot{X} - \bar{\theta}_1 \Gamma \dot{\theta}_1)^2 - \frac{g}{2} T + b\dot{c} \right) \right] \quad (2.20)$$

$$= \int \mathcal{D}[X, \theta_1, b, c] \exp \left[i \int \left(-T \sqrt{-(\dot{X} - \bar{\theta}_1 \Gamma \dot{\theta}_1)^2} + b\dot{c} \right) \right] \quad (2.21)$$

The canonical action becomes second order form action (2.20) by integrating out the canonical momentum p , and the second order form action reduces to the square root type action by integrating out the einbein g . Although there exists dynamical ghost parts, they never appear as long as physical states matrix elements are taken.

The gauge fixed action in the conformal gauge is obtained by setting $g = 1$ in the gauge fixing function in (2.16),

$$I = \int \left(p(\dot{X} - \bar{\theta}_1 \Gamma \dot{\theta}_1) - \frac{1}{2T} (p^2 + T^2) \right) \quad (2.22)$$

Then equations of motion in this gauge are calculated as

$$T\ddot{X} = 0 \quad , \quad \dot{\theta}_1 = 0 \quad . \quad (2.23)$$

Although X and θ_1 satisfy free field equations, the action can not be written in simple free form for the manifest supersymmetry and the Lorentz covariance. If the fermionic part of action is written in free form [10], coordinates transform non-covariantly under both the Lorentz and the global supersymmetry transformations.

This system has the N=2 global supersymmetry whose charges are given by

$$Q_A \epsilon_A = \zeta \epsilon - \bar{\theta}(\not{p} - T\Gamma_{11})\epsilon \quad . \quad (2.24)$$

They satisfy the SUSY algebra with the central extension given by

$$\{Q_{A,\alpha}, Q_{B,\beta}\}_D = -2(C\Sigma)_{A,\alpha B,\beta} \quad . \quad (2.25)$$

The use of the Dirac bracket does not change the algebra calculated with the Poisson bracket since Poisson brackets between global SUSY charges and fermionic constraints are zero, $\{Q_A, f_B\} = 0$. Since the right hand side of (2.25) is the same as Σ in (2.5), one half of eigenvalues is zero. If we choose the linear combination of these supercharges as

$$\begin{cases} \tilde{Q}_1 &= Q_1 \\ \tilde{Q}_2 &= Q_2 + Q\not{p}/T \end{cases} \quad , \quad (2.26)$$

their Dirac brackets become

$$\{\tilde{Q}_A, \tilde{Q}_B\}_D = -2 \begin{pmatrix} C\not{p} & 2Ch \\ 2Ch & -\frac{2}{T}C\not{p}h \end{pmatrix} \quad . \quad (2.27)$$

Here h is the constraint in (2.2). In the canonical quantization procedure the Dirac bracket is replaced by the commutator; $\{q, p\}_D = 1 \rightarrow [q, p] = i$. Using the above SUSY algebra, the BPS bound is obtained:

$$\| \tilde{Q}_2^\alpha |*\rangle \|^2 = \frac{1}{2} \langle * | [\tilde{Q}_2^\alpha, \tilde{Q}_2^\alpha]_+ |* \rangle = \frac{4}{T} p_0(p_0 - T)(p_0 + T) \| |*\rangle \|^2 \geq 0 \quad , \quad (2.28)$$

in the rest frame ($p_0 > 0$) with $C = i\Gamma_0$. The τ reparametrization constraint, $h = 0$ in (2.2), leads to that the super-D-particle is the BPS saturated state. For the BPS saturated state, $p_0 = T$, the SUSY algebra becomes

$$[\tilde{Q}_A, \tilde{Q}_B]_+ = \begin{pmatrix} 2T & 0 \\ 0 & 0 \end{pmatrix} \quad . \quad (2.29)$$

This shows that \tilde{Q}_1^α invariance is spontaneously broken and θ_1 is recognized as the Nambu-Goldstone fermion. On the other hand, \tilde{Q}_2^α invariance is preserved. In fact \tilde{Q}_2 is equivalent to the gauge generator

$$\tilde{Q}_2 = \frac{1}{T} \tilde{f}_2 - 4\bar{\theta}_1 h \quad . \quad (2.30)$$

Therefore \tilde{Q}_2 generates rather the local supersymmetry than the global supersymmetry. The super-D-particle system represents one half of the N=2 global supersymmetry generated by \tilde{Q}_1

$$\{\tilde{Q}_1, \tilde{Q}_1\} = -2C\not{p} \quad . \quad (2.31)$$

Indeed the gauge fixed actions, (2.19), (2.20) and (2.21), have manifest global supersymmetry generated by $Q_1 = \tilde{Q}_1$ as (2.31), but they do not represent another half of SUSY generated by Q_2 . The Q_2 invariance seems to contradict with the gauge fixing of the local supersymmetry, $\theta_2 = 0$: $\{\theta_2, Q_2\epsilon_2\} = \epsilon_2 \neq 0$. However it is just superficial and the use of the Dirac bracket with the correct second class constraints set resolves this. After gauge fixing the Dirac bracket is modified by taking into account the set of second class constraints, $f_1 = f_2 = \theta_2 = 0$,

$$\begin{aligned} \{A, B\}_{D2} &= \{A, B\}_D - \{A, \theta_2\} \frac{TC\not{p}C^{-1}}{p^2} \{f_1, B\} - \{A, \theta_2\} \{f_2, B\} \\ &+ \{A, f_1\} \frac{T\not{p}}{p^2} \{\theta_2, B\} - \{A, f_2\} \{\theta_2, B\} \quad . \end{aligned} \quad (2.32)$$

Hence the Q_2 transformation keeps the gauge condition $\theta_2 = 0$ invariant, $\{\theta_2, Q_2\epsilon_2\}_{D2} = \epsilon_2 - \epsilon_2 = 0$. In other words, the $\theta_2 = 0$ gauge is conserved with using the local supersymmetry as discussed in [6]. The Q_2 invariance of the gauge fixed action (2.22) can be checked explicitly by using with non-linear transformation rules

$$\begin{aligned} \delta_2\theta_1 &= \{\theta_1, Q_2\epsilon_2\}_{D2} = \frac{T\not{p}}{p^2}\epsilon_2 \quad , \quad \delta_2X = \{X, Q_2\epsilon_2\}_{D2} = T\bar{\theta}_1\Gamma\frac{\not{p}}{p^2}\epsilon_2, \\ \rightarrow \delta_2I &= \int [p(\delta_2\dot{X} - \delta_2(\bar{\theta}_1\Gamma\dot{\theta}_1))] = 2 \int \partial_0(T\bar{\theta}_1\epsilon_2) = 0 \quad . \end{aligned} \quad (2.33)$$

The situation becomes clearer by introducing the Dirac * variable

$$A^* \equiv A - \phi_a(\{\phi, \phi\})_{ab}^{-1} \{\phi_b, A\} \quad . \quad (2.34)$$

The Q_1 symmetry is realized as usual while the Q_2 symmetry is realized associating with the local supersymmetry:

$$Q_1^* = Q_1 \quad , \quad Q_2^* = Q_2 - \frac{1}{T}\tilde{f}_2 \quad . \quad (2.35)$$

The true degree of freedom of the global supersymmetry is one half of it by comparing the transformation rule δ_2 in (2.33) with δ_1 ;

$$T\frac{\not{p}}{p^2}\epsilon_2 = \epsilon_1 \quad . \quad (2.36)$$

This relation is nothing but the relation (2.30), $\tilde{Q}_2 = Q_2 + Q_1\not{p}/T \approx 0$. Therefore the BPS condition, (2.27) and (2.29), is explained as follows: In the $\theta_2 = 0$ gauge, one half of the global SUSY generated by Q_2 is a linear combination of Q_1 by using with the local SUSY generated by \tilde{f}_2 . Surviving spinor, θ_1 , represents the one half of the global SUSY generated by Q_1 manifestly.

3 Canonical analysis of the super-D-string

The action of the super-D-string [6] is given by

$$\begin{aligned} S &= -T \int d^2\sigma \sqrt{-\det(G + \mathcal{F})} + T \int \Omega_{(2)} \\ &= T \int d^2\sigma [-\sqrt{-G_F} - \epsilon^{\mu\nu} \Omega_{\mu\nu}(\tau_1)] \end{aligned} \quad (3.1)$$

where

$$\begin{aligned} G_F &= \det(G_{\mu\nu} + \mathcal{F}_{\mu\nu}) = G + \mathcal{F}_{01}^2 \quad , \quad G = \det G_{\mu\nu} = G_{00}G_{11} - G_{01}^2 \\ G_{\mu\nu} &= \Pi_\mu^m \Pi_{\nu,m} \quad , \quad \Pi_\mu^m = \partial_\mu X^m - \bar{\theta} \Gamma^m \partial_\mu \theta \\ \mathcal{F}_{01} &= \epsilon^{\mu\nu} [\partial_\mu A_\nu - \Omega_{\mu\nu}(\tau_3)] \end{aligned}$$

and

$$\begin{aligned} \Omega_{\mu\nu}(\tau_1) &= -\bar{\theta} \Gamma \tau_1 \partial_\mu \theta \cdot (\Pi_\nu + \frac{1}{2} \bar{\theta} \Gamma \partial_\nu \theta) \\ \Omega_{\mu\nu}(\tau_3) &= -\bar{\theta} \Gamma \tau_3 \partial_\mu \theta \cdot (\Pi_\nu + \frac{1}{2} \bar{\theta} \Gamma \partial_\nu \theta) \quad . \end{aligned} \quad (3.2)$$

Notations of the Dirac matrices and the N=2 spinor indices are followed from the reference [6]. The definition of the canonical momenta leads to the following primary constraints :

$$h \equiv \frac{1}{2T_E} (\tilde{p}^2 + (T_E)^2 G_{11}) = 0 \quad (3.3)$$

$$t \equiv \tilde{p} \cdot \Pi_1 = 0 \quad (3.4)$$

$$F \equiv \zeta + \bar{\theta} (\tilde{\not{p}} - \not{A}_1 \tau_E) - \frac{1}{2} (\bar{\theta} \Gamma \theta' \cdot \bar{\theta} \Gamma \tau_E + \bar{\theta} \Gamma \tau_E \theta' \cdot \bar{\theta} \Gamma) = 0 \quad (3.5)$$

$$E^0 = 0. \quad (3.6)$$

Here p_m , ζ and E^μ are the canonical momenta conjugate to X^m , θ and A_μ respectively and

$$\tilde{p} \equiv p + \bar{\theta} \Gamma \tau_E \theta' \quad . \quad (3.7)$$

The τ matrices, which act on the $N = 2$ spinor indices of θ_A with the same chirality, appear only through the combination of τ_E

$$\tau_E \equiv E^1 \tau_3 + T \tau_1 \quad , \quad \text{and} \quad \tau_E^2 = T_E^2 \quad . \quad (3.8)$$

Here T_E defined by

$$(T_E)^2 \equiv T^2 + (E^1)^2, \quad (3.9)$$

is regarded as the tension of the D-string.

The algebra of fermionic constraints

$$\{F_{A,\alpha}, F_{B,\beta}\} = 2(C\Xi)_{A,\alpha B,\beta} \quad (3.10)$$

with

$$\Xi_{AB} \equiv \tilde{p}\delta_{AB} - \mathbb{I}_1(\tau_E)_{AB} = \tilde{p}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \mathbb{I}_1\begin{pmatrix} E^1 & T \\ T & -E^1 \end{pmatrix} \quad . \quad (3.11)$$

As same as the super-D-particle case, (2.6) and (2.7), Ξ has nilpotency as a matrix

$$\Xi^2 = 2T_E h - 2\tau_E t \approx 0 \quad (3.12)$$

but each block of Ξ has non-zero determinant. It follows that the rank of Ξ is one half of 32 as long as T_E and Π_1^2 are non-zero.

The canonical Hamiltonian becomes a sum of the primary constraints in a similar form as the GS superstring [12],

$$\begin{aligned} \mathcal{H} &= \int d\sigma [p_m \dot{X}^m + \zeta_{A,\alpha} \dot{\theta}^{A,\alpha} + E^\mu \dot{A}_\mu - \mathcal{L}] \\ &= \int d\sigma [g_0 h + g_1 t + F\chi + E^0 \dot{A}_0 - (\partial_1 E^1) A_0] \\ &= \int d\sigma [g_0 \tilde{H} + g_1 \tilde{T} + F\Xi\tilde{\chi} + E^0 \dot{A}_0 - (\partial_1 E^1) A_0] \quad . \end{aligned} \quad (3.13)$$

In the second line, bosonic parameters are

$$g_0 = \frac{\sqrt{-G_F} T_E}{G_{11} T} \quad , \quad g_1 = \frac{G_{01}}{G_{11}} \quad . \quad (3.14)$$

The consistency condition of the primary constraints (3.6) leads to the U(1) Gauss law constraint,

$$\partial_1 E^1 = 0 \quad , \quad (3.15)$$

as the secondary constraint. No further condition appears. The fermionic parameter, χ , is determined by the consistency condition of F in (3.5) to be conserved under the time development. Using with (3.10), (3.11) and (3.12) the consistency condition,

$$\Xi \left[\chi + g_0 \frac{\tau_E}{T_E} \theta' - g_1 \theta' \right] = 0 \quad , \quad (3.16)$$

leads to the following solution

$$\chi = \Xi \tilde{\chi} - g_0 \frac{\tau_E}{T_E} \theta' + g_1 \theta' \quad . \quad (3.17)$$

In the last line of (3.13), \tilde{H} and \tilde{T} are the first class combinations of the bosonic constraints:

$$\tilde{H} = h - \frac{1}{T_E} F \tau_E \theta' = \frac{1}{2T_E} (p^2 + (T_E)^2 X'^2) - \frac{1}{T_E} \zeta \tau_E \theta' \quad (3.18)$$

$$\tilde{T} = t + F \theta' = p \cdot X' + \zeta \theta' \quad (3.19)$$

Now all constraints are conserved for any g_0 , g_1 and $\tilde{\chi}$. From the canonical Hamiltonian (3.13) we can also read that the first class combinations of the fermionic constraints are

$$\tilde{F} \equiv F\Xi \quad . \quad (3.20)$$

They satisfy a closed algebra,

$$\begin{aligned} \{\tilde{F}_{A,\alpha}, \tilde{F}_{B,\beta}\} &= -4[L_{AC}(C\Xi)_{CB,\alpha\beta} + ((\tilde{F}\tau_E)_{A,(\alpha}\bar{\theta}'_{B,\beta)} + \tilde{F}_{A,(\alpha}(\bar{\theta}'\tau_E)_{B,\beta)}) \\ &\quad + \frac{1}{2}(\tilde{F}\tau_E\Gamma\theta' \cdot (C\Gamma)_{AB,\alpha\beta}) + \frac{1}{2}\tilde{F}\Gamma\theta' \cdot (C\Gamma\tau_E)_{AB,\alpha\beta}) \\ &\quad + \frac{1}{8}(\partial_1 E^1)\bar{\theta}\Gamma\Xi_{A,(\alpha} \cdot \bar{\theta}\Gamma\tau_3\Xi_{B,\beta)}]\delta(\sigma - \sigma') \end{aligned} \quad (3.21)$$

with the symmetric bracket, $A_{(\alpha\beta)} = A_{\alpha\beta} + A_{\beta\alpha}$. The square of constraints are set to be zero in the Hamiltonian algebra. In the right hand side of (3.21) we have introduced combinations of the bosonic first class constraints

$$L_{AB} \equiv T_E\tilde{H}\delta_{AB} - \tau_{E,AB}\tilde{T} = \frac{1}{2}\Xi_{AB}^2 - F\tau_E\theta'\delta_{AB} - F\theta'\tau_{E,AB} \quad . \quad (3.22)$$

The above constraints, $\tilde{F} = L = 0$, form a reducible first class constraint set. Instead we can choose an irreducible and covariant set of first class constraints:

$$\begin{cases} \tilde{F}_1 &= \tilde{F}(1 + \tau_3)/2 = 0 \\ L_{11} &= T_E\tilde{H} - E^1\tilde{T} = 0 \\ L_{12} &= -T\tilde{T} = 0 \end{cases} \quad . \quad (3.23)$$

The point of this irreducibility is that “chiral” projection $\frac{1+\tau_3}{2}$ does not commute with the first class projection operator, Ξ : \tilde{F} are linearly dependent, $(\tilde{F})_\alpha C^\alpha \approx 0 \leftrightarrow C^\alpha = (\Xi)_\beta^{\alpha} \tilde{C}^\beta$. On the other hand \tilde{F}_1 are linearly independent, $(\tilde{F}_1)_\alpha C^\alpha = 0 \leftrightarrow C^\alpha = 0$. \tilde{F}_1 is the canonical generator of the local supersymmetry (kappa symmetry). The canonical projection operator Ξ in (3.11) is related to the normalized projection operator $\gamma^{(1)}$ derived by Schwarz [6] as

$$\Xi = [1 - (\frac{\tilde{p}\mathbb{M}_1}{TG_{11}}\tau_1 - \frac{E^1}{T}i\tau_2)](-T\mathbb{M}_1\tau_1) = [1 - \gamma^{(1)}](-T\mathbb{M}_1\tau_1) \quad . \quad (3.24)$$

Fermionic constraints independent of the first class constraints \tilde{F}_1 are second class;

$$F_2 \equiv F\frac{1 - \tau_3}{2} = 0 \quad . \quad (3.25)$$

The Dirac bracket defined by the second class constraints is

$$\{A, B\}_D = \{A, B\} + \{A, F_2\}\frac{\Xi_{22}C^{-1}}{2T^2G_{11}}\{F_2, B\} \quad . \quad (3.26)$$

Using the Dirac bracket, (3.26), the algebra of the first class constraints is calculated:

$$\begin{aligned} \{\tilde{F}_{1,\alpha}, \tilde{F}_{1,\beta}\}_D &= -4[L_{1A}(C\Xi)_{A1,\alpha\beta} + \frac{1}{2}E^1(2\tilde{F}_{1,(\alpha}\bar{\theta}'_{1,\beta)} + \tilde{F}_1\Gamma\theta'_1 \cdot (C\Gamma)_{\alpha\beta}) \\ &\quad + \frac{1}{2}T(2\tilde{F}_{1,(\alpha}\bar{\theta}'_{2,\beta)} + \tilde{F}_1\Gamma\theta'_2 \cdot (C\Gamma)_{\alpha\beta})]\delta(\sigma - \sigma') \end{aligned} \quad (3.27)$$

$$\{\tilde{F}_{1,\alpha}, L_{11}\}_D = -E^1(\tilde{F}_1(\sigma) + \tilde{F}_1(\sigma'))\delta'(\sigma - \sigma') \quad (3.28)$$

$$\{\tilde{F}_{1,\alpha}, L_{12}\}_D = -T(\tilde{F}_1(\sigma) + \tilde{F}_1(\sigma'))\delta'(\sigma - \sigma') \quad (3.29)$$

$$\{L_{11}, L_{11}\}_D = [-2E^1(L_{11}(\sigma) + L_{11}(\sigma')) + T(L_{12}(\sigma) + L_{12}(\sigma'))]\delta'(\sigma - \sigma') \quad (3.30)$$

$$\{L_{11}, L_{12}\}_D = -T(L_{11}(\sigma) + L_{11}(\sigma'))\delta'(\sigma - \sigma') \quad (3.31)$$

$$\{L_{12}, L_{12}\}_D = -T(L_{12}(\sigma) + L_{12}(\sigma'))\delta'(\sigma - \sigma') \quad (3.32)$$

Here in right hand sides the square of constraints as well as the Gauss law constraint are omitted for simplicity.

So far we have assumed that G_{11} is non-vanishing. If there were a solution with $G_{11} = 0$, the above discussions of covariant and irreducible separation of the first and second class constraints could not be applied. The Dirac bracket (3.26) becomes ill-defined because G_{11} enters in the denominator of the second term. Furthermore $G_{11} = 0$ is inconsistent with the static gauge, because it represents point-like state. This situation, $G_{11} = 0$, occurs in the ground state of the Green-Schwarz superstring which is an obstacle of the covariant quantization. The condition, $G_{11} \neq 0$, may be described as follows: Assuming that the inside of the square root of the DBI action (3.1) is non negative

$$-G - \mathcal{F}_{01}^2 \geq 0 \quad , \quad (3.33)$$

then in the conformal gauge, $G_{00} + G_{11} = G_{01} = 0$,

$$G_{11}^2 \geq \mathcal{F}_{01}^2 \geq 0 \quad . \quad (3.34)$$

Therefore non-vanishing \mathcal{F}_{01} could avoid $G_{11} = 0$ singularity. Since \mathcal{F}_{01} is related to the E^1 which is the canonical conjugate of A_1 by

$$E^1 \equiv T \frac{\mathcal{F}_{01}}{\sqrt{-G_F}} \quad , \quad (3.35)$$

vanishing \mathcal{F}_{01} means that the electric field, E^1 , is zero. Actually $E^1 = 0$ states are identified with the states of the Green-Schwarz superstring as will be discussed in section 5. In other words non-vanishing value of E^1 distinguishes the D-string from the Green-Schwarz superstring.

4 Covariant quantization of the super-D-string

We take the irreducible and covariant set of constraints and construct the BRST charge of the D-string. Using with the first class algebra (3.27) -(3.32) we obtain

$$Q_B = Q_{B,0} + Q_{B,1} + Q_{B,nm} \quad (4.1)$$

$$Q_{B,0} = \int L_{11}c_{11} + L_{12}c_{12} + \tilde{F}_1\gamma_1 + (\partial_1 E^1)\mu \quad (4.2)$$

$$\begin{aligned} Q_{B,1} = & \int [b_{11} (2E^1c_{11}c'_{11} + T(c_{11}c'_{12} - c'_{11}c_{12}) + 2\bar{\gamma}_1\Xi_{11}\gamma_1) \\ & + b_{12} (T(-c_{11}c'_{11} + c_{12}c'_{12}) + 2\bar{\gamma}_1\Xi_{21}\gamma_1) \\ & + E^1 (8(\beta_1\gamma_1)(\bar{\theta}'_1\gamma_1) + 2(\beta_1\Gamma\theta'_1) \cdot (\bar{\gamma}_1\Gamma\gamma_1) + (\beta_1\gamma_1)c'_{11} - (\beta_1\gamma'_1)c_{11}) \\ & + T (8(\beta_1\gamma_1)(\bar{\theta}'_2\gamma_1) + 2(\beta_1\Gamma\theta'_2) \cdot (\bar{\gamma}_1\Gamma\gamma_1) + (\beta_1\gamma_1)c'_{12} - (\beta_1\gamma'_1)c_{12})] \end{aligned} \quad (4.3)$$

$$Q_{B,nm} = \int \pi_{g,1A}c_{1A} + \pi_\psi\tilde{\gamma}_1 \quad . \quad (4.4)$$

The BRST charge (4.1) is shown to be nilpotent; $\{Q_B, Q_B\}_D = 0$. Since we take the irreducible set of constraints infinite number of additional fields are not required in the covariant formalism. Analogous to the super-D-particle case (2.12), the path integral is given by,

$$\begin{aligned} Z = & \int d\mu_{\text{can}} \delta(F_2)(\det\{F_2, F_2\})^{-1} \\ & \exp i \left[\int (p\dot{X} + \zeta_A\dot{\theta}_A + E^1\dot{A}_1 + b_{1A}\dot{c}_{1A} + \beta_1\dot{\gamma}_1 - \{\Psi, Q_B\}_D) \right] \quad , \\ & d\mu_{\text{can}} = \mathcal{D}[\zeta_A, \theta_A, p, X, A_1, E^1; c_{1A}, b_{1A}, \gamma_1, \beta_1] \quad . \end{aligned} \quad (4.5)$$

The covariant gauge condition $\theta_1 = 0$ [6] can be chosen by using local supersymmetry degree of freedom. In order to make reparametrization gauge degree of freedom manifest we keep values of g_μ unfixed and construct the gauge fixing function as

$$\Psi = \int \frac{g_0}{2T_E}b_{11} + \left(\frac{g_0}{2} \frac{E^1}{T_E} - \frac{g_1}{2}\right) \frac{1}{T}b_{12} - \beta_1\psi - \tilde{\beta}_1\theta_1 - \nu A_0 \quad . \quad (4.6)$$

The contribution from the second class constraints is also exponentiated by introducing χ_2 and η_2 . The path integral becomes

$$Z = \int d\mu_{\text{can}} \mathcal{D}[\chi_2, \eta_2, \rho_2; g, \psi, \pi_\psi, A_0, \tilde{\gamma}, \tilde{\beta}] \exp i \int \mathcal{L} \quad (4.7)$$

$$\begin{aligned} \mathcal{L} = & F_2\chi_2 + \bar{\eta}_2 \frac{(\Xi_{22})C}{2T^2G_{11}}\rho_2 + p\Pi_0 + \zeta_1\dot{\theta}_1 + F_2\dot{\theta}_2 + E^1\dot{A}_1 \\ & - g_0\tilde{H} - g_1\tilde{T} + \tilde{F}_1\psi + (E^1)'A_0 - (F_2 - \zeta_2 - \bar{\theta}_2\dot{\theta})\dot{\theta}_2 + \pi_\psi\theta_1 \\ & + b_{1A}\dot{c}_{1A} - g_0h_{gh} - g_1t_{gh} + \beta_1\dot{\gamma}_1 + \beta_1\tilde{\gamma}_1 + \tilde{\beta}_1\Xi_{11}\gamma_1 \quad , \end{aligned} \quad (4.8)$$

with

$$\begin{aligned} h_{gh} = & \frac{E^1}{2T_E} \{2(b'_{11}c_{11} + 2b_{11}c'_{11}) + b'_{12}c_{12} + 2b_{12}c'_{12}\} \\ & + \frac{T}{2T_E} (b'_{11}c_{12} + 2b_{11}c'_{12} + b'_{12}c_{11} + 2b_{12}c'_{11}) \quad , \\ t_{gh} = & -\frac{1}{2} (b'_{11}c_{11} + 2b_{11}c'_{11} + b'_{12}c_{12} + 2b_{12}c'_{12}) \quad . \end{aligned} \quad (4.9)$$

Integrating out auxiliary fermionic fields and canonical momenta it is expressed with the minimal fields measure

$$d\mu = \mathcal{D}[X, A_0, A_1, \theta_2, c_{1A}, b_{1A}] \quad . \quad (4.10)$$

The resultant path integrals are

$$Z = \int d\mu \mathcal{D}[p, E^1; g_0, g_1] \exp \left[i \int \left(\hat{p} \hat{\Pi}_0 + E^1 \hat{\mathcal{F}}_{01} - g_0 \hat{h} - g_1 \hat{t} + b_{1A} \dot{c}_{1A} - g_0 h_{gh} - g_1 t_{gh} \right) \right] \quad (4.11)$$

$$= \int d\mu \mathcal{D}[E^1; g_0, g_1] \exp \left[i \int \left(-\frac{T_E}{2g_0} (\hat{\Pi}_0 - \frac{g_1}{2} \hat{\Pi}_1)^2 - \frac{g_0}{2} T_E \hat{G}_{11} + E^1 \hat{\mathcal{F}}_{01} + b_{1A} \dot{c}_{1A} - g_0 h_{gh} - g_1 t_{gh} \right) \right] \quad (4.12)$$

$$= \int d\mu \mathcal{D}[E^1] \exp \left[i \int \left(-T_E \sqrt{-\det \hat{G}_{\mu\nu}} + E^1 \hat{\mathcal{F}}_{01} + b_{1A} \dot{c}_{1A} \right) \right] \quad (4.13)$$

$$= \int d\mu \exp \left[i \int \left(-T \sqrt{-\det(\hat{G}_{\mu\nu} + \hat{\mathcal{F}}_{\mu\nu})} + b_{1A} \dot{c}_{1A} \right) \right] \quad . \quad (4.14)$$

Where hat variables stand for $\theta_1 = 0$, $\hat{\mathcal{O}} = \mathcal{O}|_{\theta_1=0}$; for example, $\hat{\Pi}_\mu = \partial_\mu X - \bar{\theta}_2 \Gamma \partial_\mu \theta_2$ etc. In the static gauge, reparametrization ghosts disappear and the gauge fixed action is obtained as expected by Schwarz [6].

It is also interesting to perform A_μ integral in the action (4.13). The variation of U(1) gauge fields leads to that E^1 is constant

$$I = \int \left(-T_E \sqrt{-\det \hat{G}_{\mu\nu}} - E^1 \hat{\Omega}_{01}(\tau_3) + b_{1A} \dot{c}_{1A} \right) \quad . \quad (4.15)$$

This form is similar to the GS action in the classical level. In eq.(4.15) coefficients of the DBI action and the Wess-Zumino term, $\Omega(\tau_3)$, are different contrasting with the GS action in which both coefficients are T . Same coefficients follow from the local supersymmetry invariance, while different coefficients represent that the local supersymmetry is fixed. Corresponding to that the tension of the GS superstring is T , the tension of the super-D-string is T_E . The coefficient of $\Omega(\tau_3)$ for the super-D-string is E^1 and the one for the GS superstring is 1 in unit of T , so this form may manifest the NS charge coupling.

The gauge fixed action in the conformal gauge is obtained by choosing $g_0 = 1$ and $g_1 = 0$ in the gauge fixing function (4.6),

$$I = \int p \cdot (\dot{X} - \bar{\theta}_2 \Gamma \dot{\theta}_2) - \frac{1}{2T_E} (p^2 + T_E^2 X'^2) + E^1 (F_{01} - \bar{\theta}_2 \dot{X}' \dot{\theta}_2 + \bar{\theta}_2 \Gamma \dot{\theta}_2 \bar{\theta}_2 \Gamma \theta'_2) + \mathcal{L}_{gh} \quad . \quad (4.16)$$

Once the gauge fixed action is obtained, equations of motion and boundary conditions are determined. Although it was supposed to choose the conformal gauge, obtained field equations are not conformal. The reasons are the global SUSY, existence of E^1 and the

covariant gauge, $\theta_1 = 0$. One can check explicitly that the contribution of $\theta_1 = 0$ breaks conformal form in the canonical equation of motion.

Boundary conditions on the bosonic coordinates are determined by the surface term of the variation of the gauge fixed action (4.16): In the tangential directions of the brane they are

$$\delta X_\mu|_{\text{at boundary}} = 0 \quad (4.17)$$

in order to be consistent with the static gauge. In the transverse direction it may be chosen as

$$X'_i|_{\text{at boundary}} = 0 \quad (4.18)$$

from the momentum conservation. On the other hand fermionic coordinate, θ_2 , must satisfy the periodic (antiperiodic) condition in the $\theta_1 = 0$ gauge.

This system has N=2 global supersymmetry whose charges are obtained as

$$Q_A \epsilon_A = \int d\sigma [\zeta \epsilon - \bar{\theta} \not{p} \epsilon + \bar{\theta} \not{X}' \tau_E \epsilon - \frac{1}{6} \bar{\theta} \Gamma \tau_E \theta' \cdot \bar{\theta} \Gamma \epsilon - \frac{1}{6} \bar{\theta} \Gamma \theta' \cdot \bar{\theta} \Gamma \tau_E \epsilon] \quad (4.19)$$

The global supersymmetry charges make the following algebra:

$$\{Q_{A,\alpha}, Q_{B,\beta}\}_D = -2(C\Xi_G)_{A,\alpha B,\beta} + \int (\partial_1 E^1) \begin{pmatrix} 0 & \bar{\theta}_1 \Gamma_\alpha \cdot \bar{\theta}_2 \Gamma_\beta \\ -\bar{\theta}_2 \Gamma_\alpha \cdot \bar{\theta}_1 \Gamma_\beta & 0 \end{pmatrix} \quad (4.20)$$

where

$$(\Xi_G)_{AB} \equiv \not{P} - \int \not{X}' (\tau_E)_{AB} = \not{P} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \int \not{X}' \begin{pmatrix} E^1 & T \\ T & -E^1 \end{pmatrix} \quad (4.21)$$

and $P \equiv \int p$ is the total momentum. The second term in (4.20) is the Gauss law constraint, (3.15). We assumed that the surface term vanishes by the boundary condition. It is important to notice that E^1 is constant by the equation of motion and the constraint (3.15) so τ_E is also constant. According to Polchinski [1], the SUSY algebra shows the NS-NS charge and the R-R charge of the super-D-string as $(E^1, 1)$. This is consistent with the result obtained by SL(2,Z) representation of the tension [6].

There is a bases in which the supersymmetry algebra (4.20) is diagonalized

$$\begin{cases} \tilde{Q}_1 &= \frac{1}{\sqrt{2T_E(T_E - E^1)}} (TQ_1 + (T_E - E^1)Q_2) \Gamma_1 C^{-1} \\ \tilde{Q}_2 &= \frac{1}{\sqrt{2T_E(T_E - E^1)}} (TQ_2 - (T_E - E^1)Q_1) \Gamma_1 C^{-1} \end{cases}, \quad (4.22)$$

in a rest frame where $\int C \not{X}' = l C \Gamma_1$. Anti-hermiticity of $C \Gamma_1$ leads to imaginary eigenvalues. The SUSY algebra becomes

$$\{\tilde{Q}_A, \tilde{Q}_B\}_D = -2i \begin{pmatrix} P_0 - lT_E & 0 \\ 0 & P_0 + lT_E \end{pmatrix}. \quad (4.23)$$

Analogous to the previous section, the canonical quantization procedure replaces the Dirac bracket by the quantum operator bracket; $\{q, p\}_D = 1 \rightarrow [q, p] = i$, and the norm of the quantum states leads to the BPS bound:

$$\| \tilde{Q}_1 |* \rangle \|^2 = \frac{1}{2} \langle * | [\tilde{Q}_1, \tilde{Q}_1]_+ |* \rangle = (P_0 - lT_E) \| |* \rangle \|^2 \geq 0 \quad . \quad (4.24)$$

For the BPS saturated state the SUSY algebra would be

$$[\tilde{Q}_A, \tilde{Q}_B] = \begin{pmatrix} 0 & 0 \\ 0 & 4T_E l \end{pmatrix} \quad . \quad (4.25)$$

This shows that one half of the global SUSY is spontaneously broken.

For the super-D-string states, the mass operator comes from the zero-mode of constraints \tilde{H} in eq.(3.18),

$$\int d\sigma \tilde{H}(\sigma) = \frac{1}{2T_E} \left[\frac{l}{l^2} P^2 + l(T_E)^2 + \text{higher mode} \right] = 0 \quad . \quad (4.26)$$

The higher mode contribution is positive semi-definite which would be obtained by using remaining constraints. This condition leads to

$$\frac{l}{l^2} P^2 + l(T_E)^2 \leq 0 \quad \rightarrow \quad P_0 \geq T_E l \quad , \quad (4.27)$$

i.e. the ground state of the super-D-string is BPS saturated state. Then the ground state of super-D-string is a representation of the half of the N=2 global supersymmetry.

The superficial breaking down of one half of the global SUSY is also caused by the gauge fixing condition, and which is recovered by the non-linear SUSY transformation analogous to the super-D-particle case (2.33). The new Dirac bracket is introduced by $F_1 = F_2 = \theta_1 = 0$,

$$\begin{aligned} \{A, B\}_{D2} &= \{A, B\}_D - \{A, \theta_1\} \{F_1, B\} - \{A, F_1\} \{\theta_1, B\} \\ &\quad + \{A, \theta_1\} (\Xi_{12}) (\Xi_{22})^{-1} \{F_2, B\} + \{A, F_2\} (\Xi_{22})^{-1} (\Xi_{21}) \{\theta_1, B\} \end{aligned} \quad (4.28)$$

where $\{, \}_{D2}$ is defined in (3.26). The non-linear transformation rules generated by Q_1 are given by

$$\begin{cases} \delta \theta_2 &= \{ \theta_2, Q_1 \epsilon_1 \}_{D2} = \Xi_{22}^{-1} \Xi_{21} \epsilon_1 \\ \delta X &= \{ X, Q_1 \epsilon_1 \}_{D2} = \bar{\theta}_2 \Gamma \Xi_{22}^{-1} \Xi_{21} \epsilon_1 \end{cases} \quad . \quad (4.29)$$

The Dirac * variables are calculated as

$$Q_1^* = Q_1 - \int \tilde{F}_1 \frac{\tilde{\mathcal{M}} - E^1 \mathbb{M}_1}{-T^2 G_{11}} \quad , \quad Q_2^* = Q_2 \quad . \quad (4.30)$$

Under the covariant gauge, $\theta_1 = 0$, Q_2 is realized usually while Q_1 is realized by using with the local supersymmetry, \tilde{F}_1 . The global SUSY parameters are related as

$$\epsilon_2 = \Xi_{22}^{-1} \Xi_{21} \epsilon_1 = \left(\frac{E^1}{T} + \frac{i \mathbb{M}_1}{T G_{11}} \right) \epsilon_1 \quad , \quad (4.31)$$

if they are constants. Then the true degree of freedom of the global SUSY becomes one half. In fact the ground state of the D-string with $G_{11} \neq 0$, N=2 reduces to N=1. This is consistent with the BPS condition.

5 Relation between the super-D-string and the Green-Schwarz superstring

The constraint set of the Green-Schwarz superstring has the same form as those of the D-string, (3.3), (3.4) and (3.5). The former is obtained by replacements $\tau_E \rightarrow T_E \tau_3$ and $T_E \rightarrow T$. At first let us analyze the procedure of $\tau_E \rightarrow T_E \tau_3$ systematically. We consider a U(1) rotation, $U = e^{i\tau_2 \frac{\psi}{2}}$, mixing τ_1 and τ_3 :

$$\begin{aligned}\tau_3 &\rightarrow U^t \tau_3 U = \tau_3 \cos \psi + \tau_1 \sin \psi \equiv \tau(\psi) \\ \tau_1 &\rightarrow U^t \tau_1 U = \tau_1 \cos \psi - \tau_3 \sin \psi = \tau(\psi + \frac{\pi}{2})\end{aligned}\quad . \quad (5.1)$$

We define the ψ -dependent Wess-Zumino term, $\Omega_{(2)}(\psi)$

$$\Omega_{(2)}(\psi) \equiv \bar{\theta} \tau(\psi) \Gamma_m d\theta (dX^m + \frac{1}{2} \bar{\theta} \Gamma^m d\theta) \quad , \quad (5.2)$$

which is a surface term of a super symmetric closed three form, $I_{(3)}(\psi) = d \Omega_{(2)}(\psi)$ for any ψ . The Wess-Zumino action for the GS superstring is $\Omega_{(2)}(0)$, while the one for the super-D-string is $\Omega_{(2)}(\pi/2)$. We can construct one parameter family of models with the same symmetry as the super-D-string, whose action is

$$I = -T \int d^2\sigma \sqrt{-\det(G + \mathcal{F})} + T \int \Omega_{(2)}(\psi + \pi/2) \quad (5.3)$$

$$\mathcal{F} = dA + \Omega_{(2)}(\psi) \quad .$$

It is invariant under the local supersymmetry (kappa symmetry) for any value of ψ , in addition to the reparametrization invariance and the super Poincare symmetry. The super-D-string is the case for $\psi = 0$ and $\mathcal{F} \neq 0$ and the GS superstring is the case for $\psi = -\pi/2$ and $\mathcal{F} = 0$.

The U transformation is also interpreted as the SO(2) rotation of two spinors θ_1 and θ_2 , $\theta_A \rightarrow U_{AB} \theta_B$. The boundary condition $\theta_1 = 0$ for the super-D-string is obtained by $\psi = \pi/2$ rotation from the boundary condition of the GS superstring, $\theta_1 - \theta_2 = 0$. For the super-D-string right(left) handedness of fermionic coordinates are mixed because of the central extension of the SUSY algebra. The SO(2) rotation with $\psi_0 = \tan^{-1}(T_E - E^1)/T$ makes fermionic coordinates θ_A to be right/left handed eigenstates; in the constraint algebra (3.21) the bosonic constraint L_{AB} becomes diagonal. The SO(2) rotates the global SUSY algebra also, so it seems that SO(2) relates the NS-NS charge and the R-R charge when they are identified with the diagonal element and off-diagonal element of SUSY central charges respectively. This degeneracy occurs due to the flatness of the target space. By taking into account background with a dilaton ϕ and an axion χ , the SUSY central charges becomes SL(2,R) covariant representation. For the flat background the SUSY central charges for a unit string length is essentially

$$(\tau_E)_{AB} = (E^1, T) \begin{pmatrix} \tau_3 \\ \tau_1 \end{pmatrix}_{AB} \quad (5.4)$$

in (4.21). With the background it becomes

$$\begin{aligned} (\tau_{E,\text{bg}})_{AB} &= (E^1, T) e^{\phi/2} \begin{pmatrix} 1 & 0 \\ -\chi & e^{-\phi} \end{pmatrix} \begin{pmatrix} \tau_3 \\ \tau_1 \end{pmatrix}_{AB} \\ &\equiv (q_{NS}, q_R) K \begin{pmatrix} \tau_3 \\ \tau_1 \end{pmatrix}_{AB}, \end{aligned} \quad (5.5)$$

where K is a representation of $\text{SL}(2, \mathbb{R})/\text{SO}(2)$ satisfying

$$K^T \mathcal{M} K = 1, \quad \mathcal{M} = e^{\phi} \begin{pmatrix} |\lambda|^2 & \chi \\ \chi & 1 \end{pmatrix}. \quad (5.6)$$

Therefore $K \begin{pmatrix} \tau_3 \\ \tau_1 \end{pmatrix}$ transforms as a $\text{SL}(2, \mathbb{R})$ doublet when $\begin{pmatrix} \tau_3 \\ \tau_1 \end{pmatrix}$ does as a $\text{SO}(2)$ doublet. $K \begin{pmatrix} \Omega(\psi) \\ \Omega(\psi + \pi/2) \end{pmatrix}$ corresponds to $\begin{pmatrix} B_{\mu\nu}^{NS} \\ B_{\mu\nu}^R \end{pmatrix}$ in the Green-Schwarz formalism. The $\tau_{E,\text{bg}}$ in (5.5) is $\text{SL}(2, \mathbb{R})$ covariant if we regard (E^1, T) as a $\text{SL}(2, \mathbb{R})$ (contravariant) doublet.

The relation between the action (5.3) and the manifest S-dual action [14] is interesting. It is introduced a "dual U(1) field \tilde{A} " as a $\text{SL}(2, \mathbb{R})$ partner of the U(1) field A , and \tilde{E}^1 plays a role of T in (5.5). The action (5.3) is flat case of manifest the S-dual action [14]. Indeed the combination of the momenta \tilde{p} in (3.7) is a remnant of $\text{SO}(10) \times \text{SL}(2, \mathbb{R})$ as (2.8) in [14].

Next let us examine the condition $T_E \rightarrow T$ for the super-D-string to coincide with the GS superstring. The difference between the Green-Schwarz superstrings and the super-D-string is the value of E^1 in the constraints set (3.3), (3.4) and (3.5), while it is the value of \mathcal{F}_{01} in the action level (5.3). Since E^1 is the canonical conjugate of A_1 as (3.35), both $E^1 = 0$ and $\mathcal{F}_{01} = 0$ are same conditions. As seen in (3.34) these conditions allow existence of the point-like ground state. There is another situation for the super-D-string where the above condition is approximately satisfied; There are so highly excited states of the super-D-string where \mathcal{F} can be neglected. These states behave similar to highly excited states of the GS superstring. Then the covariantly quantizable super-D-string inspires the covariantly quantizable GS superstring. However at the particle-like state each elements of the projection operator in (3.11) become nilpotent, $\Xi_{AB} \rightarrow \not{p} \delta_{AB}$. Then neither covariant separation of fermionic constraints nor well-defined Dirac bracket can be constructed. The SUSY algebra for the massless point ground state becomes

$$\{Q_A, Q_B\}_D = -2C\mathcal{P}\delta_{AB} \rightarrow -2P_+ \begin{pmatrix} C\Gamma_- & 0 \\ 0 & C\Gamma_- \end{pmatrix}. \quad (5.7)$$

Each element of the above matrix is nilpotent and it is difficult to find the covariant Nambu-Goldstone fermions. The light-cone variables as Nambu-Goldstone fermions are relatively convenient. Therefore the covariant quantization of the Green-Schwarz superstring is still open problem because of the massless particle ground state.

6 Conclusions

We performed the canonical analysis of super-D-brane actions for $p=0$ and 1. Irreducible and covariant separation of fermionic constraints into the first and second classes can be possible thanks to off-diagonal elements of the fermionic constraints algebra, Σ in (2.5) for $p=0$ and Ξ in (3.11) for $p=1$. Off-diagonal elements give mass to ground states of D-branes. In the canonical analysis of the super-D-string the singularity at $G_{11} = 0$ appears where the covariant separation and the Dirac bracket become ill-defined. A sufficient condition to avoid this singularity is $\mathcal{F} \neq 0$. This condition also guarantees static picture of it. Non-zero \mathcal{F} means the $U(1)$ excitation on the world sheet of the D-brane (string) which is caused by strings whose ends are on the brane. The physical situation corresponding to $\mathcal{F} = d\mathcal{A} + \Omega_{(2)} \neq 0$ should be clarified more.

The BRST charges for the super-D-particle and the super D-string are calculated and covariant gauge fixed actions are derived. Since we have an irreducible set of constraints infinite number of fields are not required in contrast to the covariant quantization of the superparticle and the superstring theories. Equations of motion are examined after gauge fixing: For the super-D-particle X and θ satisfy free field equations, but the gauge fixed action can not be written in free form because of the manifest global supersymmetry. For the super-D-string X and θ do not satisfy conformal equations not only because of the central extension of the SUSY algebra and existence of E^1 but also because of the covariant gauge $\theta_1 = 0$.

The global supersymmetry algebra leads to the BPS bound. The super-D-particle and the ground state of the super-D-string are BPS saturated states. In the BPS saturated state, one half of the global supersymmetry is spontaneously broken while another half is trivial. The point is that this BPS condition can be written covariantly, so that the Nambu-Goldstone fermion is covariant spinor. This covariance reflects covariance of the gauge fixing. In the action one half of the global SUSY is realized manifestly, while another half is realized trivially. The use of the Dirac starred variable clarified that trivial global SUSY generator is nothing but the local SUSY (kappa symmetry) generator.

As results of checking physical picture of the super-D-string action (3.1) for $\mathcal{F} \neq 0$ ($E^1 \neq 0$): (1) It has the NS-NS charge and the R-R charge, $(q_{NS}, q_R) = (E^1, 1)$. (2) The ground state of the super-D-string is the BPS saturated state. (3) The D-string tension is scaled as $T \rightarrow T_E = \sqrt{(E^1)^2 + T^2} = \sqrt{q_{NS}^2 + T^2 q_R^2}$. (4) It is consistent with the static gauge.

We also clarified the relation between the Green-Schwarz superstring and the super D-string. It turns out that the local supersymmetric D-string action can be parametrized by a $SO(2)$ parameter, ψ , as (5.3). The GS superstring is the case for $\psi = -\pi/2$ with $\mathcal{F} = 0$, while the super-D-string is the case for $\psi = 0$ with $\mathcal{F} \neq 0$. This $SO(2)$ mixes up the right/left handedness, so it changes the boundary condition of spinor coordinates. This $SO(2)$ also acts on the global SUSY algebra. By taking into account a dilaton and an axion, the SUSY central charges become $SL(2, \mathbb{R})$ representation. In contrast with an approach to realize $SL(2, \mathbb{R})$ manifestly [14], the action (3.1) does not have manifest

SL(2,R) symmetry since the dual U(1) field is fixed. The action (3.1) describes superstrings with charges $(q_{NS}, q_R) = (E^1, 1)$ except $E^1 = 0$, where singularity appears then the string reduces to a fundamental superstring. Therefore we suggest that rather the value of E^1 plays a role of the order parameter of the action (3.1); the super-D-string exists for $E^1 \neq 0$ while the fundamental GS superstring exists for $E^1 = 0$. Further study is necessary for the super-D-string physics.

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